数理解析研究所講究録
第 1615 巻 2008 年 28-33

QUICK REVIEW ON PROPERTY (X)

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ABSTRACT. We will review some materials that are useful to prove the uniqueness of preduals. Those were used crucially in our recent work on the uniqueness of predual of any 'finite' non-commutative H^{∞}.

1. INTRODUCTION

In [12] we established, among other things, the uniqueness of predual of any 'finite' non-commutative H^{∞}-algebra $H^{\infty}(M,\tau)$, which was introduced by Bill Arveson modeled after the usual pair $H^{\infty}(\mathbb{D}) \hookrightarrow L^{\infty}(\mathbb{T})$ with the aid of operator algebra theory. The class of finite non-commutative H^{∞}-algebras contains $H^{\infty}(\mathbb{D})$ as well as its abstract generalizations. Thus [12, Theorem 2] covers any existing generalization of the famous result due to Tsuyoshi Ando [3].

The most key ingredient of our proof of the uniqueness of predual of $H^{\infty}(M,\tau)$ is to provide a non-commutative analog of Amar–Lederer's peak set result [2] (also see [4]), which we fully explained in [12]. However, our proof of the uniqueness of predual also uses two purely Banach space theoretic techniques – Property (X) due to Godefroy and Talagrand and a very clever trick, both of which we just borrowed from some references without any detailed explanation. Here we will give detailed accounts (for non-experts like us) on those techniques as supplements to [12, Theorem 2].

In closing, we should mention our sincere thanks to Professor Kichi-Suke Saito for giving this opportunity.

2. GODEFROY–TALAGRAND'S PROPERTY (X)

This section mainly follows Gedefroy and Talagrand's elegant work [6]. The key ingredient behind Godefroy–Talagrand's property (X) is the next proposition.

Proposition 2.1. *Let E and G be Banach spaces with $E^{\star} = G^{\star}$. If a sequence $\{x_n\} \subset E^{\star}$ satisfies*

(i) $x_n \longrightarrow 0$ *in* $\sigma(E^{\star},E)$; *and*
(ii) $\sum_{n=1}^{\infty} |\psi(x_{n+1}-x_n)| < +\infty$ *for all* $\psi \in E^{\star\star}$,

then $x_n \longrightarrow 0$ *in* $\sigma(E^{\star},G)$.

Proof. Set $u_0 := x_1$, $u_1 := x_2 - x_1$, and $u_n := x_{n+1} - x_n$, and then by (i)

$$\sum_{k=0}^{n} u_k = x_{n+1} \longrightarrow 0 \quad \text{in } \sigma(E^{\star},E). \tag{1}$$

Date: 9/20/2008.
AMS subject classification: Primary: 46B20; secondary: 46L52.

For each $n \in \mathbb{N}_0 := \{0,1,2,\dots\}$ we consider the map $T_n : \alpha = (\alpha_k) \in \ell^\infty(\mathbb{N}_0) \mapsto \sum_{k=0}^n \alpha_k u_k \in E^\star$ ($\hookrightarrow E^{\star\star\star}$ via the canonical embedding). Then one has, by (ii),

$$\sup\{|(T_n\alpha)(\phi)| : \|\alpha\|_\infty \le 1,\, n \in \mathbb{N}_0\} \le \sum_{k=0}^{\infty} |\phi(u_k)| < +\infty$$

for all $\phi \in E^{\star\star}$, and hence the uniform boundedness principle shows that there is $K > 0$ such that

$$\left\| \sum_{k=0}^{n} \alpha_k u_k \right\|_{E^\star} = \|T_n \alpha\|_{E^{\star\star\star}} \le K \tag{2}$$

for all $n \in \mathbb{N}_0$ and for all $\alpha_k \in \mathbb{C}$ with $|\alpha_k| \le 1$.

Choose an arbitrary free ultrafilter $\omega \in \beta(\mathbb{N}_0) \setminus \mathbb{N}_0$ and put $\xi_\omega := \lim_{n\to\omega} \sum_{k=0}^n u_k$ in $\sigma(E^\star, G)$. Let us choose arbitrary $n_1 < n_2 < \cdots < n_{2l-1} < n_{2l}$. Then, using (2) with

$$\alpha_k = \begin{cases} 1 & n_{2j-1} \le k \le n_{2j},\ j = 1,\dots,l, \\ 0 & \text{otherwise} \end{cases}$$

we get

$$\left\| \sum_{k=n_1}^{n_2} u_k + \sum_{k=n_3}^{n_4} u_k + \cdots + \sum_{k=n_{2l-1}}^{n_{2l}} u_k \right\| \le K.$$

Here we have

$$\begin{aligned} \sum_{k=n_1}^{n_2} u_k + \sum_{k=n_3}^{n_4} u_k + \cdots + \sum_{k=n_{2l-1}}^{n_{2l}} u_k &= \sum_{k=n_1}^{n_2} u_k + \sum_{k=n_3}^{n_4} u_k + \cdots + \left(\sum_{k=0}^{n_{2l}} u_k - \sum_{k=0}^{n_{2l-1}} u_k \right) \\ &\longrightarrow \sum_{k=n_1}^{n_2} u_k + \sum_{k=n_3}^{n_4} u_k + \cdots + \xi_\omega - \sum_{k=0}^{n_{2l-1}} u_k \quad \text{in } \sigma(E^\star, G) \end{aligned}$$

as $n_{2l} \to \omega$ but $n_1, \dots, n_{2l-1}$ are fixed. Then it follows that

$$\left\| \sum_{k=n_1}^{n_2} u_k + \sum_{k=n_3}^{n_4} u_k + \cdots + \xi_\omega - \sum_{k=0}^{n_{2l-1}} u_k \right\| \le K$$

for any fixed $n_1 < n_2 < \cdots < n_{2l-1}$. We also have, by (1),

$$\begin{aligned} &\sum_{k=n_1}^{n_2} u_k + \sum_{k=n_3}^{n_4} u_k + \cdots + \xi_\omega - \sum_{k=0}^{n_{2l-1}} u_k \\ &\longrightarrow \sum_{k=n_1}^{n_2} u_k + \sum_{k=n_3}^{n_4} u_k + \cdots + \xi_\omega - 0 \quad \text{in } \sigma(E^\star, E) \end{aligned}$$

as $n_{2l-1} \to \infty$ but $n_1, \dots, n_{2l-2}$ are fixed. Therefore, we get

$$\left\| \sum_{k=n_1}^{n_2} u_k + \sum_{k=n_3}^{n_4} u_k + \cdots + \sum_{k=n_{2l-3}}^{n_{2l-2}} u_k + \xi_\omega \right\| \le K$$

for any fixed $n_1 < n_2 < \cdots < n_{2l-2}$. Clearly, this procedure can be continued for n_{2l-2}, n_{2l-4} and so on, and we finally get $l \cdot \|\xi_\omega\| = \|l\xi_\omega\| \le K$. Since l can be arbitrarily large, ξ_ω must be zero for any $\omega \in \beta(\mathbb{N}_0) \setminus \mathbb{N}_0$, which means that $\lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} \sum_{k=0}^n u_k = 0$ in $\sigma(E^\star, G)$. ☐

Based on the lemma, Godefroy and Talagrand introduced property (X).

Definition 2.1. A Banach space E has property (X) if for any $\psi \in E^{**}$ the following conditions are equivalent:

(a) $\psi \in E$ with the canonical embedding $E \hookrightarrow E^{**}$.
(b) For any sequence $\{x_n\} \subset E^*$ with the properties
  - $x_n \longrightarrow 0$ in $\sigma(E^*, E)$,
  - $\sum_{n=1}^{\infty} |\phi(x_{n+1} - x_n)| < +\infty$ for all $\phi \in E^{**}$,

  one has $\psi(x_n) \longrightarrow 0$.

This definition gives, in some sense, a criterion of w^*-continuity for bounded linear functionals on the dual E^* of a Banach space E with property (X).

Definition 2.2. A Banach space E is said to be the unique predual of its dual E^* if another Banach space G with $G^* = E^*$ must coincide with E inside the dual E^{**} of E^* ($= G^*$) via the canonical embedding.

Corollary 2.2. *If a Banach space E has property* (X)*, then E must be the unique predual of its dual E^*.*

Proof. Assume another Banach space G satisfies $G^* = E^*$. Embed $G \hookrightarrow (E^*)^* = E^{**}$ by $g(x) := x(g)$ for $x \in E^* = G^*$ and $g \in G$. Let $\{x_n\} \subset E^*$ be chosen in such a way that $x_n \longrightarrow 0$ in $\sigma(E^*, E)$ and $\sum_{n=1}^{\infty} |\phi(x_{n+1} - x_n)| < +\infty$ for all $\phi \in E^{**}$. By Proposition 2.1 we get $x_n \longrightarrow 0$ in $\sigma(E^*, G)$, which shows that $g(x_n) = x_n(g) \longrightarrow 0$ for all $g \in G$. Thus, Property (X) ensures that any g must fall in $E \hookrightarrow E^{**}$, that is, $G \subseteq E$ inside E^{**}. If $G \subsetneq E$ inside E^{**}, then by the Hahn–Banach extension theorem there is $x \in E^*$ such that $x \neq 0$ but $x|_G = 0$. (Indeed, there is $e \in E \setminus G$ by the assumption, and thus $[e] \in E/G$ with $[e] \neq 0$. Then by the Hahn–Banach extension theorem there is $\varphi \in (E/G)^*$ sending $[e]$ to $\|[e]\| = \inf\{\|e - g\| : g \in G\} \neq 0$. Hence the $x := \varphi \circ Q \in E^*$ with the quotient map $Q : E \to E/G$ becomes a desired element.) This x is a non-zero element in $G^* = E^*$ but it is identically zero on G, a contradiction. Hence $G = E$ inside E^{**}. □

The next proposition has been known, but we do give one proof, which is a prototype of our proof of the uniqueness of predual of $H^\infty(M, \tau)$.

Proposition 2.3. *Let M be a σ-finite von Neumann algebra and M_* be its predual. Then, M_* has property* (X)*.*

Proof. It suffices to show that, if $\varphi \in M^*$ satisfies $\varphi(x_n) \longrightarrow 0$ for any $\{x_n\} \subset M$ with the properties

- $x_n \longrightarrow 0$ in $\sigma(M, M_*)$ and
- $\sum_{n=1}^{\infty} |\phi(x_{n+1} - x_n)| < +\infty$ for all $\phi \in M^*$,

then φ must fall in $M_* \hookrightarrow M^*$. Here we need the following standard facts on von Neumann algebras (see e.g. [9] and [11] for their proofs):

(1) Any $\psi \in M^*$ can be decomposed into $\psi = \psi_{\rm nor} + \psi_{\rm sing}$ with $\psi_{\rm nor} \in M_*$ and $\psi_{\rm sing} \in M^* \ominus M_*$, and $\|\psi\| = \|\psi_{\rm nor}\| + \|\psi_{\rm sing}\|$ holds. (This is the so-called *non-commutative Lebesgue decomposition* due to Takesaki.) We call M_* the normal part and $M^* \setminus M_*$ the singular part. Remark that the notation here is a little bit different from that in [12].
(2) For any $\psi \in M^*$ (or $\psi \in M_*$) there are a unique positive linear functional $|\psi| \in M_*$ (resp. $|\psi| \in M_*$) and a unique partial isometry $v \in M^{**}$ (resp. $v \in M_*$) such that $\langle \psi, x \rangle = \langle |\psi|, xv \rangle$ as well as $\langle |\psi|, x \rangle = \langle \psi, xv^* \rangle$ for $x \in M^{**}$, where $\langle \cdot, \cdot \rangle : M^* \times M^{**} \to \mathbb{C}$ stands for the canonical pairing. (This is the so-called *polar decomposition*

of linear functionals due to Sakai and also Tomita.) Remark here that the second dual M^{**} becomes a von Neumann algebra, which naturally contains the original M as a subalgebra via the canonical embedding $M \hookrightarrow M^{**}$.

(3) Both the closed subspaces $M_\star$ and $M^\star \ominus M_\star$ of $M^\star$ are closed under the operation $\psi \in M^\star \mapsto |\psi| \in M^\star$. (This follows from the construction of the decomposition in (1) together with (2).)

(4) For a positive linear functional $\psi \in M^\star$ the following are equivalent:
- $\psi \in M^\star \ominus M_\star$.
- For every nonzero projection $e \in M$ there is a non-zero projection $e_0 \in M$ such that $e_0 \leq e$ and $\psi(e_0) = 0$.

(This is Takesaki's criterion for 'singularity' of linear functionals.)

(5) Any $\psi \in M^\star$ (or $M_\star$) can be written as a linear combination of four positive linear functionals in $M^\star$ (resp. $M_\star$).

Let us decompose the given φ into $\varphi = \varphi_{\rm nor} + \varphi_{\rm sing}$ as in (1), and what we have to show is $\varphi_{\rm sing} = 0$, i.e., $\varphi = \varphi_{\rm nor} \in M_\star$. For contrary we suppose $\varphi_{\rm sing} \neq 0$. Then, by (2) and (3), $|\varphi_{\rm sing}| \neq 0$ and $|\varphi_{\rm sing}| \in M^\star \ominus M_\star$ still holds. Clearly, the orthogonal families of non-zero projections in $\mathrm{Ker}|\varphi_{\rm sing}|$ forms an inductive set by inclusion, and Zorn's lemma ensures the existence of a maximal family $\{q_k\}$, which is at most countable since M is σ-finite. Put $q_0 := \sum_k q_k$ in M, and then $q_0 = 1$ since $q_0 \neq 1$ clearly contradicts to the above (4). Also, if $\{q_k\}$ is a finite family, then $|\varphi_{\rm sing}|(1) = \sum_k |\varphi_{\rm sing}|(q_k) = 0$, a contradiction. Therefore, $\{q_k\}$ must be a countably infinite family with $\sum_k q_k = 1$ in M. Letting $p_n := 1 - \sum_{k\leq n} q_k$ we have $p_n \searrow 0$ in $\sigma(M, M_\star)$ but $|\varphi_{\rm sing}|(p_n) = |\varphi_s|(1)$ for all n. The latter says that p_n converges a non-zero projection $p \in M^{**}$ in $\sigma(M^{**}, M^\star)$ with $\langle |\varphi_{\rm sing}|, p\rangle = \langle |\varphi_{\rm sing}|, 1\rangle$ $(= |\varphi_{\rm sing}|(1))$ since p_n is a decreasing sequence. Let $u \in M$ and $v \in M^{**}$ be the partial isometries for the polar decompositions of $\varphi_{\rm nor}$ and $\varphi_{\rm sing}$, respectively. Then, for $x \in M^{**}$ one has $|\langle\varphi_{\rm sing}, (1-p)x\rangle| = |\langle|\varphi_{\rm sing}|, (1-p)xv\rangle| \leq \langle|\varphi_{\rm sing}|, 1-p\rangle^{1/2}\langle|\varphi_{\rm sing}|, v^*x^*xv\rangle^{1/2} = 0$ so that $\langle\varphi_{\rm sing}, x\rangle = \langle\varphi_{\rm sing}, px\rangle$ since $\langle|\psi_{\rm sing}|, p\rangle = \langle|\psi_{\rm sing}|, 1\rangle$. Similarly, for $x \in M^{**}$ one has $|\langle\varphi_{\rm nor}, px\rangle| = |\langle|\varphi_{\rm nor}|, pxu\rangle| \leq \langle|\varphi_{\rm nor}|, p\rangle^{1/2}\langle|\varphi_{\rm nor}|, u^*x^*xu\rangle^{1/2}$. Since $|\varphi_{\rm nor}|$ still falls in $M_\star$, $\langle|\varphi_{\rm nor}|, p\rangle = \lim_{n\to\infty}|\varphi_{\rm nor}|(p_n) = 0$ so that $\langle\varphi_{\rm nor}, px\rangle = 0$. Consequently, we get $\langle\varphi, px\rangle = \langle\varphi_{\rm nor}+\varphi_{\rm sing}, px\rangle = \varphi_{\rm sing}(x)$ for $x \in M$.

Let $x \in M$ be arbitrary. Clearly, $p_n x \longrightarrow 0$ in $\sigma(M, M_\star)$. Let $\phi \in M^\star$ be arbitrary, and decompose $y \in M \mapsto \phi(yx)$ into a linear combination of four positive linear functionals $\phi_i \in M^\star$, $i = 1, 2, 3, 4$, thanks to the above (5). Since $\sum_{n=1}^N |\phi_i(p_{n+1} - p_n)| = \sum_{n=1}^N \phi_i(q_{n+1}) = \phi_i(\sum_{n=2}^{N+1} q_n) \leq \phi_i(1) < +\infty$ for all $N \in \mathbb{N}$, it follows that $\sum_{n=1}^\infty |\phi(p_{n+1}x - p_n x)| < +\infty$. Therefore, by the assumption here one has $\varphi(p_n x) \longrightarrow 0$. On the other hand, $\varphi(p_n x) = \langle\varphi, p_n x\rangle \longrightarrow \langle\varphi, px\rangle = \varphi_{\rm sing}(x)$ so that $\varphi_{\rm sing} = 0$, a contradiction. □

The heart of the above proof is as follows. Although $\varphi_{\rm nor}$ and $\varphi_{\rm sing}$ are 'orthogonal', we cannot find a projection in M that distinguishes those. (Of course, we can find such a projection in M^{**} since both functionals can be regarded as 'normal' ones on M^{**}.) Thus we first construct a projection $p \in M^{**}$ in such a way that it can be 'nicely' approximated by projections in M and p is greater than 'the support of $\varphi_{\rm sing}$' but 'disjoint' from 'the support of $\varphi_{\rm nor}$'. This essentially says that M 'remembers' the decomposition '$M^\star = M_\star \oplus (M^\star \ominus M_\star)$' of $M^\star$ (the second dual of $M_\star$). This suggests us that such a decomposition of the second dual should be related to property (X) of a Banach space in question. This was quite recently answered affirmatively by Hermann Pfitzner when a Banach space in question is separable, see [8].

Further accounts on the present topics can be found in [5].

3. Addendum – a clever trick due to Pełczyński

The essential idea of our proof of the uniqueness of predual of $H^\infty(M,\tau)$ is similar to that of Proposition 2.3. However, the luck of self-adjointness of our algebra $H^\infty(M,\tau)$ (thus we cannot use the order structure) makes some trouble, which we overcame with a clever trick borrowed from the proof of [7, Proposition 1.c.3]. (The trick is due to Aleksander Pełczyński, see [10, p.637] for this credit, and it was originally used for proving that if a Banach space has Pełczyński's property (u) then so does any closed subspace, see [7] or more recent [1].) Here we will explain it. The situation we deal with is as follows. Let M be a von Neumann algebra and A be its σ-weakly closed (possibly non-self-adjoint) unital subalgebra. Assume that we have two sequences $\{a_n\}\subset A$ and $\{b_n\}\subset M$ such that

(i) both a_n and b_n converge to the same $p\in M^{**}$ in $\sigma(M^{**},M^\star)$, and
(ii) $\sum_{n=1}^\infty|\phi(b_{n+1}-b_n)|<+\infty$ for all $\phi\in M^\star$.

What we want to do is to replace a_n by a new one with keeping (i) and further satisfying (ii). This can be done by utilizing the above-mentioned clever trick in Banach space theory.

Proposition 3.1. *There is another $\{a'_n\}\subset A$ such that*

(i') $a'_n\longrightarrow p$ in $\sigma(M^{**},M^\star)$, and
(ii') $\sum_{n=1}^\infty|\phi(a'_{n+1}-a'_n)|<+\infty$ for all $\phi\in M^\star$.

We need one elementary lemma due to Stanisław Mazur.

Lemma 3.2. *Let E be a normed space and $\{x_n\}\subset E$ be such that $x_n\longrightarrow 0$ in $\sigma(E,E^\star)$. Then, for each $\varepsilon>0$ and each $m\in\mathbb{N}$ there is a convex combination $y=\sum_{n\geq m}\lambda_n x_n$ with $\|y\|<\varepsilon$.*

Proof. Let C_m be the closed convex hull of $\{x_n\}_{n\geq m}$ in E. It suffice to show $0\in C_m$. Thus, for contrary, suppose $0\notin C_m$. Then there is a small open ball B centered at 0 with $C_m\cap B=\emptyset$. The Hahn–Banach separation theorem ensures that there are $\varphi\in E^\star$ and $t\in\mathbb{R}$ such that $\mathrm{Re}\varphi(b)\lneqq t\leqq\mathrm{Re}\varphi(c)$ for all $b\in B$ and $c\in C_m$. This is impossible since $x_n\longrightarrow 0$ in $\sigma(E,E^\star)$ (implying $t\leq 0$) and $0\in B$ (implying $t\gneqq 0$). Thus $0\in C_m$, which means the desired assertion. □

Proof. (Proposition 3.1) Putting $b_0:=0$ we have $\sum_{n=1}^\infty|\phi(b_n-b_{n-1})|<+\infty$ for all $\phi\in M^\star$. Set $u_n:=a_n-\sum_{k=1}^n b_k-b_{k-1}$, and then $u_n=a_n-b_n\longrightarrow 0$ in $\sigma(M,M^\star)$ by (i). By Lemma 3.2 there are convex combinations $u'_j=\sum_{n=p_{j-1}+1}^{p_j}\lambda_n^{(j)}u_n$ such that $0=p_0<p_1<p_2<\cdots$ and $\|u'_j\|\leq 2^{-j}$. Then We define $a'_j:=\sum_{n=p_{j-1}+1}^{p_j}\lambda_n^{(j)}a_n\in A$ and put $a'_0:=0$ for convenience. Let us prove that this $\{a'_j\}$ gives a desired sequence.

Since $a_n\longrightarrow p$ in $\sigma(M^{**},M^\star)$, for any $\varepsilon>0$ and any $\phi\in M^\star$ there is $n_0\in\mathbb{N}$ such that $|\langle a_n,\phi\rangle-\langle p,\phi\rangle|<\varepsilon$ for all $n\geq n_0$, where $\langle\cdot,\cdot\rangle:M^{**}\times M^\star\mapsto\mathbb{C}$ is the canonical pairing. If j_0 is chosen so that $p_{j_0-1}+1\geq n_0$, then one has $|\langle a'_j,\phi\rangle-\langle p,\phi\rangle|\leq\sum_{n=p_{j-1}+1}^{p_j}\lambda_n^{(j)}|\langle a_n,\phi\rangle-\langle p,\phi\rangle|<\varepsilon$ for all $j\geq j_0$. Thus $a'_j\longrightarrow p$ in $\sigma(M^{**},M^\star)$ as $j\to\infty$.

One has

$$\begin{aligned}
a'_{j+1}-a'_j&=u'_{j+1}+\sum_{n=p_j+1}^{p_{j+1}}\lambda_n^{(j+1)}(a_n-u_n)-u'_j-\sum_{n=p_{j-1}+1}^{p_j}\lambda_n^{(j)}(a_n-u_n)\\
&=u'_{j+1}-u'_j+\sum_{n=p_j+1}^{p_{j+1}}\lambda_n^{(j+1)}(\sum_{k=1}^n b_k-b_{k-1})-\sum_{n=p_{j-1}+1}^{p_j}\lambda_n^{(j)}(\sum_{k=1}^n b_k-b_{k-1})\\
&=u'_{j+1}-u'_j+\sum_{n=p_{j-1}+1}^{p_{j+1}}\mu_n^{(j)}(b_n-b_{n-1})
\end{aligned}$$

with some $0 \leq \mu_n^{(j)} \leq 1$. Hence,

$$\begin{aligned}
&\sum_{j=0}^{\infty} |\phi(a'_{j+1} - a'_j)| \\
&\leq \sum_{j=0}^{\infty} \|\phi\| \|u'_{j+1}\| + \sum_{j=0}^{\infty} \|\phi\| \|u'_j\| + \sum_{j=1}^{\infty} \sum_{n=p_{j-1}+1}^{p_{j+1}} \mu_n^{(j)} |\phi(b_n - b_{n-1})| \\
&\leq 2 \sum_{j=0}^{\infty} \|\phi\| \|u'_j\| + \sum_{n=1}^{\infty} |\phi(b_n - b_{n-1})| \\
&\leq 4\|\phi\| + \sum_{n=1}^{\infty} |\phi(b_n - b_{n-1})| < +\infty
\end{aligned}$$

by $\|u'_j\| \leq 2^{-j}$ and (ii). □

Remark here that the argument presented above uses only the linear structure; hence clearly it can be applied to more general situations.

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